P. H. Souto Ribeiro[1*], S. Pádua[2], and C. H. Monken[2]

[1] *Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, RJ 22945-970, Brazil*

[2] *Departamento de Física, Universidade Federal de Minas Gerais, Caixa Postal 702, Belo Horizonte, MG 30123-970, Brazil*

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We show that the first experiment with double-slits and twin photons detected in coincidence can be understood as a quantum eraser. The "which path" information is erased by transverse indistinguishability obtained by means of mode filtering in the twin conjugated beam. A delayed choice quantum eraser based on the same scheme is proposed.

42.50.Ar, 42.25.Kb

I. INTRODUCTION

Studying Quantum Physics experimentally has become much easier with the use of the twin photons produced in the spontaneous parametric down-conversion process. Many of the experiments performed so far, are realizations of different kinds of interferometers. These interferometers allow us to improve our understanding of the nature of light and matter. It is convenient to think these interferometers, as divided in two categories: the ones utilizing the longitudinal degrees of freedom of the field and the ones utilizing its transverse degrees of freedom.

We call longitudinal interferometers, the ones of the type of Michelson's and Mach-Zehnder's interferometers. A large number of experiments with twin photons has been performed with them. Some examples are those of Refs. [1–3]. The transverse interferometers, are the ones of the kind of the double-slit or Young interferometers. Some examples are given in Refs. [4–10]. These interferometers are important because they are perfect realizations of some *gedankenexperiments*, used in the discussion of important issues on the foundations of Quantum Mechanics. The transverse interferometers are very robust, quite stable and phase differences can be controlled by simple displacement of detectors in most of the cases. For this reason, they have become an important tool in the field of multiparticle interferometry.

In this paper, we analyze a transverse interferometer utilizing the twin photons of the down-conversion. As far as we are concerned, it was the first interferometer with twin photons and a double-slit [5]. The main result of that work, was to show that interference fringes were observed in the coincidence counting rate, while at the same time, intensity interference fringes could not be observed. Our aim here is to revisit this experiment and show that it can be viewed as a quantum eraser [11]. Even though other experiments performed in the past could also be interpreted in the same way as a quantum eraser, this experiment was probably one of the first utilizing twin photons. The fact that twin photons were employed stresses the quantum character of the *which path* information erasure, since it is performed by quantum state projection of a nonlocal wave function.

II. THE DOUBLE-SLIT EXPERIMENT WITH TWIN PHOTONS

Let us briefly recall the main idea behind the experiment described in Ref. [5]. Fig. 1 shows a sketch of the set-up. Twin photons are produced in the process of spontaneous parametric down-conversion and they are detected in coincidence. The signal photon is passed through a double-slit and the idler photon goes straight to the detector. Coincidence fringes are detected by displacing transversely the signal detector, which is after the slits and keeping the idler detector fixed. It was shown that the coincidence profile exhibited interference fringes, even when intensity fringes were not observed.

FIG. 1. Two-slit interference experiment with twin photons.

As it is known, and it was demonstrated for the parametric down-conversion [4], the second order coherence, which defines the visibility of the intensity fringes, is dependent on the geometrical properties of the light source. If we have a small source, approximately spherical waves are generated and if the slits are far enough, the second order coherence is almost perfect. Then, intensity interference fringes are obtained in a double-slit experiment with unity visibility. If we have an extended source instead, the degree of coherence can be smaller than one, as well as the visibility of the intensity interference fringes. The degree of coherence depends strongly on the source length and the visibility of the fringes depends on the separation between slits and the distance between source and slits.

However, if twin photons are used and coincidence detection is performed, the coherence function is now a fourth order one. The degree of coherence becomes dependent on the way the idler beam is detected. The coincidence interference fringes also become dependent on it. In particular, it was shown [5] that the detection through a small idler aperture leads to an improvement on the coincidence fringes visibility and a large idler aperture tends to destroy the interference fringes. Even if we do not have second order interference, fourth order coherence can be obtained by the use of a small area detector for the idler beam. This is the principle of a quantum eraser.

III. THE QUANTUM ERASER ALGORITHM

For the sake of simplicity, we will demonstrate the quantum erasure aspect of the above mentioned experiment by comparing it with a simpler system. We will introduce a quantum erasure algorithm. Any experience following this procedure, can be understood as a quantum erasure. The analysis is the same as in reference [12].

FIG. 2. Quantum eraser algorithm.

The quantum eraser algorithm has three steps:

1. A superposition of state is produced and a phase dependent measurement is performed on the state. See Fig. 2a for example. A small light source emits a photon. The photon passes through a double-slit. The slits are labeled 1 and 2 and the state of the photon after the slits is given by:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle). \tag{1}$$

   By detecting the photon after the slits, in different positions in a plane transverse to the propagation direction, a phase dependent measurement is performed. The phase difference between states $|\psi_1\rangle$ and $|\psi_2\rangle$ is proportional to the path difference from each slit to the detector. Interference fringes are then observed.

2. The second step is to provide a possibility of having *which path* information. In Fig. 2b, this is performed by placing wave plates in front of each slit, so that if the photon passes through slit 1, it will be L (left) circularly polarized and if it passes through slit 2, it will emerge R (right) circularly polarized. As it is known, the mere possibility of having *which path* information destroys the interference pattern. In terms of the state of the system, states $|\psi_1\rangle$ and $|\psi_2\rangle$ have been entangled with an internal degree of freedom (the polarization), which is used to identify the path:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle|L\rangle + |\psi_2\rangle|R\rangle). \tag{2}$$

   Notice that a polarization independent detection leads to an incoherent sum of the intensities due to each polarization. This leads to no interference.

3. Finally, the third and last step consists of erasing the *which path* information. In Fig. 2c, passing the photon through a linear polarizer performs the erasure. This is equivalent to a projection of the system onto a state which is not entangled with the polarization anymore:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)|\theta\rangle, \tag{3}$$

   where θ represents the angle of the polarizer. The interference fringes are recovered.

The above state can be understood as the state of the system just before detection. In fact, it is the passage of the photon through the polarizer, which projects its state onto a linear polarization state.

In next section, we will show that the experiment of reference [5] follows the above algorithm.

IV. QUANTUM ERASURE BY TRANSVERSE INDISTINGUISHABILITY

A sequence of steps associated with the experiment of Fig. 1 is now shown in Fig. 3. In Fig. 3a, we focus on the signal beam where the double-slit plane is far enough from the down-conversion crystal, so that it emits approximately like a point source. From the detector's point of view, a photon that is detected at a particular point P on the detection plane has a probability amplitude $\psi_1(P, P')$ associated with its passage through slit 1, assuming that it was generated at point P' on the source plane. Analogously, $\psi_2(P,P')$ is the amplitude associated with generation at point P', passage through slit 2 and detection at point P. For a fixed detection point P, $\psi_1(P,P')$ and $\psi_2(P,P')$ are the diffraction amplitudes of the slits 1 and 2, respectively, on the source plane, as if they were illuminated by a point source located at P. For the configuration just described, since the source is far away from the slits, the overlap between $\psi_1(P,P')$ and $\psi_2(P,P')$ is much larger than the source dimensions. So, passage through slit 1 or slit 2 are indistinguishable possibilities and interference arises. The state of the signal photon after the slits is given by:

$$|\psi\rangle_s = \frac{1}{\sqrt{2}}(|\psi_1\rangle_s + |\psi_2\rangle_s). \tag{4}$$

The first step is accomplished.

FIG. 3. Quantum erasure by transverse indistinguishability.

Now, if we move the slits screen close to the crystal, $\psi_1(P,P')$ and $\psi_2(P,P')$ do not overlap anymore (Fig. 3*b*). That is, passage through slit 1 (2) implies that the photon was generated in region A (B) of the source. Since the interaction volume in spontaneous parametric down-conversion behaves like an incoherent source [4], emission from regions A and B are, in principle, distinguishable events. No interference is observed in this configuration. The state of the photon after the slits is now labeled by the emitting regions A and B:

$$|\psi\rangle_s = \frac{1}{\sqrt{2}}(|\psi_1\rangle_{A,s} + |\psi_2\rangle_{B,s}). \tag{5}$$

We are going to make use of the idler photon, which was not shown in Fig. 3a, to obtain the *which path* information. It was shown by Klyshko [13] that signal and idler photons in spontaneous parametric down-conversion have a high spatial correlation on the source, that is, they are generated in the same point in the interaction region. The idler photon will be labeled by $A(B)$ if it is emitted at the region $A(B)$ of the crystal. Then the state of the twin photons is given by:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle_s|A\rangle_i + |\psi_2\rangle_s|B\rangle_i), \tag{6}$$

where the index s stands for the signal beam and i for the idler. Step two is accomplished. Note that now, the system is entangled with an auxiliary system, the idler photon, which serves as a *label*.

The field generated by an incoherent source can be viewed as a statistical ensemble of coherent fields [14]. It is known from diffraction theory that in a transverse plane located in the far zone, the spatial distribution of each realization of this ensemble is proportional to the Fourier transform of its distribution on the source plane. By inverting the Fourier transform, for example with the help of a lens, an image of the source is formed in the far zone, and it is possible to detect which zone (A or B) of the source the idler photon was emitted from.

Let us now see how this path information can be erased. If a small aperture is placed on the idler detection plane, it will act as a spatial filter. In other words, it will band-limit the Fourier transform of each coherent realization of the ensemble. The smaller the aperture, the narrower the spatial bandwidth of the filtered field. Since the details of an image are carried by the high frequency components of its angular spectrum [14], the strong spatial filtering produced by a small enough aperture will make impossible to retrieve spatial information about the source. Then, this spatial filtering erases *which path* information from the point of view of the idler detector.

Expanding states $|A\rangle_i$ and $|B\rangle_i$ in terms of their transverse Fourier components we have

$$\begin{aligned} |A\rangle_i &= \int \mathcal{A}(\boldsymbol{q})\ |\boldsymbol{q}\rangle_i\ d\boldsymbol{q}\,; \\ |B\rangle_i &= \int \mathcal{B}(\boldsymbol{q})\ |\boldsymbol{q}\rangle_i\ d\boldsymbol{q}. \end{aligned} \tag{7}$$

Detection of the idler photon through a small pinhole corresponds to a projection of the states $|A\rangle_i$ and $|B\rangle_i$ onto approximately the same state $|\boldsymbol{q}_o\rangle$. $\boldsymbol{q}_o$ represents a given transverse wave vector component [14]. In the limit of a point detector, it is exactly the same state vector. Then, the state of the twin pair becomes:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle_s + |\psi_2\rangle_s)|\boldsymbol{q}_o\rangle_i. \tag{8}$$

Interference is retrieved by performing coincident detection of signal and idler photons (Fig. 3*c*). Step three is accomplished.

Again, the above state describes the photon pair just before detection, and it is the passage of the idler photon through the pinhole which projects the system onto that state. When a large aperture is used for the detection of the idler photon, the interference disappears again. The larger the detection area is, the larger is the number of Fourier components that are taken. In the limit where all Fourier components are detected, states $|A\rangle_i$ and $|B\rangle_i$ can be completely reconstructed. The state of the system is again given by expression (6) and the idler photon again serves as a label.

In the usual discussions about the quantum eraser, the state of the which-path detector is described in a two-dimensional Hilbert space. To accomplish erasure, that state is projected onto a superposition of two “pointer” states, such that a symmetric superposition gives rise to “fringes”, whereas a antisymmetric one gives rise to “antifringes” [11]. In the context discussed here, the state of the idler photon (the which-path detector) is described by a continuum of modes (its Fourier decomposition). Instead of “fringes” and “antifringes”, we have a continuum of interference patterns labeled by the selected transverse Fourier component $\boldsymbol{q}_o$.

V. A DELAYED CHOICE QUANTUM ERASER SCHEME

FIG. 4. Delayed choice quantum eraser.

The experimental set-up described in Fig. 1 can be easily changed, in order to obtain a delayed choice quantum eraser. See Fig. 4. By introducing a beam splitter in the path of the conjugated beam and sending each half to two different detectors, it is possible to choose between interference and non-interference. One of the detectors has a small aperture and the other a large aperture. These detectors and beam splitters can be put very far from the crystal, so that the signal photon passes through the double-slit before the idler photon gets to the beam splitter. A photon count in the large aperture detector, implies in no interference and a photon count in the small aperture detector implies in interference. The decision between interference and no interference is made after the photon has passed through the slits.

In the original experiment, the detection through large and small apertures is not performed simultaneously. However, there are no reasons for the results to be different. In fact, we note that in this scheme, the decision of interference or not is not made in the beam splitter, but in the detector. Finally, it is worth mentioning that our double-slit quantum eraser satisfies all the criteria for a true quantum eraser as stated by Kwiat, Steinberg and Chiao [15]: a delayed choice scheme can be implemented, it employs single particles and the distinguishing information is carried separately from the interfering particle.

VI. CONCLUSION

We have analyzed a double-slit interference experiment from a point of view of a quantum eraser. We show that the measurements performed, followed a certain quantum eraser algorithm. We also show that a delayed choice quantum eraser could be easily obtained by the insertion of a beam splitter on the idler beam and detection with different detection areas. This experiment, was probably the first realization of a quantum eraser dealing with the transverse degrees of freedom of the field.

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* Corresponding author.
E-mail address: phsr@if.ufrj.br

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